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6 SPECIALLY STRUCTURED MATHEMATICAL PROGRAMMING PROBLEMS,

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Co-Principal Investigators

10 Richard W. Cottle

[redacted]

Frederick S. Hillier

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FINAL REPORT

AFOSR Contract F44620-74-C-0079

June 1, 1974 to August 31, 1978

The Systems Optimization Laboratory (SOL) is a major component of the Department of Operations Research of Stanford University. The SOL carries on an integrated program of theoretical research, algorithmic development, and sophisticated software production for the optimization of large-scale systems, including specially structured mathematical programming problems. This program was partially supported by this contract.

The contract provided direct support to two of the SOL Co-Investigators, Professors Richard W. Cottle and Frederick S. Hillier, who served as Co-Principal Investigators for this contract. Two half-time research assistants, selected each year from among the Ph.D. candidates in the Department of Operations Research, also were supported.

Professor Cottle's work was in the area of complementarity theory, including attention to solution of variational inequality problems by constructing approximations to such problems, reducing them to large scale linear complementarity problems with special structure, and then solving them in this form. He also supervised six students — Richard S. Sacher, Robert E. Doherty, III, Ikuyo Kaneko, Jong-Shi Pang, Mark S. Goheen, and Muhamed Aganagic — who completed their Ph.D. during this period.

Professor Hillier's research program was in integer programming, including algorithmic development, decomposition methods, heuristics, and software development. Three of his students working in this area — Kevin J. Reardon, Paul F. McCoy, and Gary A. Kochman — received their Ph.D. during the term of this contract.

Abstracts of the technical reports issued under this contract are attached.

Kevin J. Reardon, "A Multi-Stage Model for Capital Budgeting with Uncertain Future Investment Opportunities," Technical Report SOL 74-9, August 1974, 48 pages.

One application of dual-angular integer programming which has received considerable attention is in the area of multi-stage capital budgeting. Research in this area is concerned with one of the most important decisions for any economic unit, public or private -- that of allocating its limited financial resources in a manner which best supports the attainment of its goals. Nearly always, such decisions must be made in an environment characterized by incomplete information, uncertainty, complex interactions among activities, imperfect capital markets, and many other complicating factors. Explicit consideration of these factors and the influence on current decisions of partial results from preceding ones often leads to complex problems with large numbers of integer decision variables.

Many mathematical programming approaches to capital budgeting have relied on assumptions that all investment opportunities which will become available within the planning horizon can be identified at the outset. Through the use of additional constraints and decision variables, this assumption is relaxed in our problem formulation to one which only requires that those future opportunities which may arise be identified, and that probabilities be assigned to the events that they become available. This approach leads to rather large numbers of diagonal submatrices in the model's dual-angular constraint matrix. The author has developed a decomposition method for solving such dual-angular integer programs; a complete discussion of this method is presented in a referenced report.

Kevin J. Reardon, "A Decomposition Method for the Solution of Dual-Angular Integer Programs," Technical Report SOL 74-10, August 1974, 80 pages.

Integer linear programming problems whose constraint matrices have dual-angular structures arise in several types of applications, particularly those which seek to specify a strategy for future actions based on observed results of previous decisions. An implicit enumeration method for integer linear programming is developed for application to problems with such dual-angular constraint matrices. The method employs a pre-solution relaxation which allows decomposition of the given problem into groups of smaller, less difficult subproblems. Reimposition of the relaxed requirements is accomplished automatically by the enumerative solution procedure. The method has been implemented within the framework of a penalty-based zero-one integer programming algorithm, and has enjoyed considerable success in reducing the solution effort for a variety of test problems. A detailed discussion of this specific implementation and its associated computational results is presented.

Ikuyo Kaneko, "On the Unboundedness of the Set of Integral Points in a Polyhedral Region," Technical Report SOL 74-12, September 1974, 21 pages.

Let $X = \{x : Ax \geq b,\ x \geq 0,\ x : \text{integral}\}$, where A and b are a given m by n matrix and m vector, respectively. This paper studies some properties concerning the unboundedness of X. The properties are in terms of (i) the existence of a "discrete ray in X" and (ii) the nonemptiness of the sets X and $\{x : Ax \geq 0,\ x \geq 0\} - \{0\}$. It is also shown that if A is rational, then (i) or (ii) characterizes the unboundedness of X.

Paul Franklin McCoy, "An Optimal Algorithm for the Resource Constrained Project Scheduling Problem," Technical Report SOL 75-26, October 1975, 173 pages.

The report presents an algorithm for solving a form of the resource constrained project scheduling problem. This particular form of the problem differs from that usually considered in that the time needed to complete a job depends on the amount of resources applied to that job. Jobs are preemptable and the objective is to minimize the project duration. It is shown that this problem is equivalent to the problem of finding that transportation polytope, defined by the resource constraints, of minimal dimension which has a face specified by the precedence constraints. A theorem is presented which gives conditions under which a face of a specified type exists. Using this theorem, the problem transforms into an integer programming problem with variables representing the completion times for each job. The constraint set is defined by inequalities involving addition and maximum operations on the variables and, without the constraint that the variables be integer, the constraint set forms a nonconvex, polyhedral set.

It is shown that an optimal solution exists in the set of points which form the peaks on the underside of the constraint set. A necessary condition that a point belong to this set is that it be a fixed point of a certain operator. This operator can be used to calculate these points and has the nice property of transforming schedules into schedules with job completion times which are less than or equal to those of the original schedule.

Two solution procedures are presented. Both start with a feasible schedule and iteratively improve upon it. One procedure is based on a branch and bound search over the fixed points of the operator mentioned above and the other procedure is based on a heuristic search over the same set of points.

The solution procedures were programmed and applied to ten problems which were derived from practical construction project scheduling problems and from problems used to test other optimal solution procedures. The number of jobs ranged from 5 to 39 and the number of resource types from 1 to 3. An optimal solution was obtained in less than half a second of computation time for seven of the smaller problems. For the other three problems both procedures improved upon the starting schedule.

Ted Eschenbach and Robert C. Carlson, "The Capacitated Multi-Period Location-Allocation Problem," Technical Report SOL 75-27, October 1975, 19 pages.

The problem is that of locating capacitated plants at some of a number of potential locations, so as to minimize the discounted sum of fixed costs incurred for open plants and variable supply costs. Both the costs and demands may vary arbitrarily over time. The results of optimal LIFO branch-and-bound and of heuristic algorithms using various branching variable selection rules and some new bounds are presented.

Richard W. Cottle and Mark S. Goheen, "A Special Class of Large Quadratic Programs," Technical Report SOL 76-7, April 1976, 75 pages.

This report is a detailed study of a class of quadratic programs having a special, rather simple structure but very large size. For the most part, these quadratic programs have strictly convex minimands and variables constrained only by upper and lower bounds. Actually, the programs studied

are even more special: the objective functions have nonpositive mixed partial derivatives. What prevents them from being trivial is their very large size, running to many thousands of variables.

Problems of the sort described above are first discussed from the "theoretical" standpoint. Their solutions are characterized by combining Kuhn-Tucker theory with their special matrix-theoretic properties. The heart of the report is a survey of potentially applicable algorithms including a new one which is believed to have superior efficiency. The report closes with a sketch of some applications of these special quadratic programs and a sample of the computational experience acquired with implementations of many of the algorithms discussed.

Jong-Shi Pang, "On a Class of Least-Element Complementarity Problems," Technical Report SOL 76-10, June 1976, 43 pages.

The present paper studies the linear complementarity problem of finding vectors x and y in R^n_+ such that $c + Dx + y \geq 0$, $b - x \geq 0$ and $x^T(c + Dx + y) = y^T(b - x) = 0$ where D is a z-matrix and $b \geq 0$. Complementarity problems of this nature arise, for example, from the minimization of certain quadratic functions subject to upper and lower bounds on the variables. Two least-element characterizations of solutions to the above linear complementarity problem are established first. Next, a new and direct method to solve this class of problems, which depends on the idea of "least-element solution" is presented. Finally, applications and computational experience with its implementation are discussed.

Gary A. Kochman, "Computer Programs for Decomposition in Integer Programming," Technical Report SOL 76-20, September 1976, 162 pages.

This report gives documentations for two computer codes, DSLC and DMLC, for solving block angular integer programming problems. The first code, DSLC, is for the single linking constraint case and the second, DMLC, is for the multiple linking constraint case.

Gary A. Kochman, "Decomposition in Integer Programming," Technical Report SOL 76-21, September 1976, 155 pages.

Linear programming models in which the constraint matrix has a block angular structure arise frequently in many applications. While much work has been devoted to exploiting this special structure when the problem variables are assumed to be continuous, little consideration has been given to models of this type in which the variables are required to take on only integer values. In this report, an algorithm for the decomposition of block angular integer programs is presented.

The block angular integer program consists of several subproblems which would operate independently except that they are tied together by a set of linking constraints. Conceptually, these linking constraints are viewed as representing common resources which the subproblems must share. The problem thus becomes that of determining an optimal allocation of these resources among the subproblems.

Towards this end, a branch-and-bound search routine is developed. It is shown how the LP-optimal dual multipliers and any slacks which appear in the optimal integer solutions to the subproblems can be used to guide the search, as well as for bounding and fathoming purposes. Special structures which arise when there is only a single linking constraint are discussed in detail.

Since the problem decomposes completely once an allocation of the linking resources is specified, only the subproblems ever need be solved explicitly. Computational results obtained with the decomposition algorithm are reported.

Gary A. Kochman, "On a Class of Concave-Separable Integer Programs," Technical Report SOL 76-22, September 1976, 14 pages.

A class of nonlinear integer programs is introduced. Problems in this class are characterized by a concave and separable objective function subject to a set of linear constraints.

It is shown how by suitably modifying the objective function, the theory of separability in linear programming can be applied to derive efficient solution procedures for problems falling in this class. This work unifies and extends several results previously obtained independently in the literature. Two illustrative applications are discussed in some detail, and specific algorithms are presented for these examples.

Mark S. Goheen, "A Decomposition Principle for Single Constraint Quadratic Programs," Technical Report SOL 77-2, January 1977, 25 pages.

This report is a study of a decomposition principle for a class of quadratic programming problems having a single coupling equality constraint. Two examples of the application of this principle are discussed in detail. The first is a portfolio selection problem, and the second a data smoothing problem dealing with direct yields of fission processes.

In both applications, it is possible to solve the decomposition subproblems in such a way that rapid solution of the main problem is possible. Some numerical experience illustrating the efficiency of this approach is reported.

Frederick S. Hillier, "A Further Investigation of Efficient Heuristic Procedures for Integer Linear Programming with an Interior," Technical Report SOL 77-13, February 1977, 61 pages.

This paper presents the results of an extensive investigation of algorithmic heuristic procedures for general pure integer linear programming problems having only inequality constraints. Included are a number of promising new variations and extensions of the procedures previously proposed by the author. Extensive computational experimentation has largely succeeded in identifying a flexible package of the most effective approaches, ranging from a very fast streamlined procedure to a very powerful combination of procedures. These procedures are both extremely efficient (comparable to the simplex method) and very effective in identifying good solutions (often obtaining an optimal one).

Although they are designed primarily for dealing algorithmically with the frequently encountered problems that are too large to be computationally feasible for exact algorithms, they also can be valuable on smaller problems by quickly providing an advanced starting solution for such algorithms.

Bruce H. Faaland and Frederick S. Hillier, "Interior Path Methods for Heuristic Integer Programming Procedures," Technical Report SOL 77-14, February 1977, 33 pages.

This paper considers heuristic procedures for general mixed integer linear programming with inequality constraints. It focuses on the question of how to most effectively initialize such procedures by constructing an "interior path" from which to search for good feasible solutions. These paths lead from an optimal solution for the corresponding linear programming problems (i.e., deleting integrality restrictions) into the interior of the feasible region for this problem. Previous methods for constructing linear paths of this kind are analyzed from a statistical viewpoint, which motivates a promising new method. These methods are then extended to piecewise linear paths in order to improve the direction of search in certain cases where constraints that are not binding on the optimal linear programming solution become particularly relevant. Computational experience is reported.

Richard W. Cottle, "Fundamentals of Quadratic Programming and Linear Complementarity," Technical Report SOL 77-21, August 1977, 27 pages.

The fundamental theory and algorithms of quadratic programming and linear complementarity are presented in expository form. Computational experience is reviewed.

Richard Asmuth, B. Curtis Eaves, and Elmor L. Peterson, "Computing Economic Equilibria on Affine Networks with Lemke's Algorithm," Technical Report SOL 77-23, September 1977, 17 pages.

Economic equilibria between supply and demand on certain affine multicommodity networks are characterized as solutions to a linear complementarity problem to which Lemke's algorithm is applied.

Muhamed Aganagic and Richard W. Cottle, "On Q-Matrices," Technical Report SOL 78-9, 19 pages.

This paper concerns the class of Q-matrices M characterized by the property that the linear complementarity problem (q,M) has a solution for every vector q. We prove that the set of nondegenerate Q-matrices is closed in the space of nondegenerate matrices, and provide a generalization of this result for degenerate matrices. Further, we prove a finite characterization of Q-matrices with nonnegative principal minors which was developed in the author's earlier report [1], and improve upon a uniqueness result of Eaves [6] for these matrices.

Muhamed Aganagic, "Iterative Methods for Linear Complementarity Problems," Technical Report SOL 78-10, September 1978, 56 pages.

This paper is concerned with analysis of the linear complementarity problem, and with methods for its solution. Several algorithms are presented, including two that apply when the matrices involved are (1) diagonally dominant, or (2) symmetric and positive semidefinite. It is shown that problems with nonsymmetric positive definite matrices can be solved by use of a relaxation technique. Furthermore, a concept analogous to that of matrix splitting is developed for the linear complementarity problem. Computational results for the proposed algorithms are presented, which indicate their potential usefulness in solving large, structured problems.

Muhamed Aganagic, "Variational Inequalities and Generalized Complementarity Problems," Technical Report SOL 78-11, September 1978, 31 pages.

This paper presents a study of the generalized complementarity problem and its relation to variational inequality problem. Although some existence results are included, the main objective of the paper is an extension of an algorithm described in [1] to the generalized complementarity problems and variational inequalities involving strongly monotone mappings.

Muhamed Aganagic, "On Diagonal Dominance in Linear Complementarity," Technical Report SOL 78-12, September 1978, 17 pages.

This paper is a study of the linear complementarity problems with diagonally dominant matrices having nonnegative diagonals. The main results include necessary and sufficient conditions for existence of solutions to the problems, as well as an efficient decomposition approach for solving them.

Daniel Granot and Frieda Granot, "Generalized Covering Relaxation for 0-1 Programs," Technical Report SOL 78-16, June 1978, 21 pages.

We construct in this paper a general purpose algorithm for solving polynomial 0-1 programming problems. The algorithm is applied directly to the polynomial problem in its original form. Further, no additional variables are introduced in the solution process.

The algorithm was tested on randomly generated modest size problems and the preliminary computational results obtained are very encouraging.

Frieda Granot, "Efficient Heuristic Algorithms for Positive 0-1 Polynomial Programming Problems," Technical Report SOL 78-20, August 1978, 24 pages.

We consider in this paper the positive 0-1 polynomial programming (PP) problem of finding a 0-1 n-vector x that maximizes $c^T x$ subject to $f(x) \leq b$ where $c, b \geq 0$ and f is an m-vector of polynomials with non-negative coefficients.

Two types of heuristic methods for solving PP problems were developed. The various algorithms were tested on randomly generated problems of up

to 1000 variables and 200 constraints. Their performance in terms of computational time and effectiveness was investigated. The results were extremely encouraging. Optimal solutions were consistently obtained by some of the heuristic methods in over 50% of the problems solved. The effectiveness was on the average better than 99% and no less than 96.5%. The computational time using the heuristic for PP problems is on the average 5% of the time required to solve the problems to optimality.

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This final report summarizes the most important results obtained under this contract. This research was carried on in the Systems Optimization Laboratory as part of an integrated program of theoretical research, algorithmic development, and sophisticated software production for the optimization of large-scale systems. The emphasis here was on theoretical research and algorithmic development for specially structured mathematical programming problems in the areas of complementarity theory and integer programming.

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